This chapter introduces a new framework for macroprudential analysis using a risk-adjusted balance sheet approach that supports policy efforts aimed at mitigating systemic risk from linkages between institutions and the extent to which they precipitate or amplify general market distress. In this regard, a forward-looking framework for measuring systemic solvency risk using an advanced form of contingent claims analysis (CCA), known as “Systemic CCA,” is presented for the systemwide capital assessment in top-down stress testing. The magnitude of joint default risk of multiple financial institutions falling into distress is modeled as a portfolio of individual market-implied expected losses (with individual risk parameters) calculated from equity market and balance sheet information. An example of Systemic CCA applied to the U.K. banking sector delivers useful insights about the magnitude of systemic losses in the context of macroprudential stress testing. In addition, the framework also helps quantify the individual contributions of institutions to systemic risk of the financial sector during times of stress.

**METHOD SUMMARY**

**Overview**
The Systemic Contingent Claims Analysis (Systemic CCA) model extends the traditional risk-adjusted balance sheet model (based on contingent claims analysis) to analyze systemic risk from the interlinkages among institutions, including time-varying dependence of default risk. This approach helps quantify the contribution of individual institutions to systemic (solvency) risk and to assess spillover risks from the financial sector to the public sector (and vice versa). In addition, Systemic CCA can be applied to macroprudential stress testing using the historical dynamics of market-implied expected losses.

**Application**
The method is appropriate in situations where access to prudential data is limited but market information is readily available.

**Nature of approach**
Model-based (option pricing, multivariate parametric/nonparametric estimation, credit default swap [CDS] curve pricing).

**Data requirements**
- Accounting information (amounts/maturities) of outstanding liabilities.
- Market data on equity and equity option prices.
- Various market rates and macro data for satellite model underpinning the stress test.
- Sovereign CDS term structure/debt level and debt repayment schedule (if financial sector-sovereign spillover risk is tested).

**Strengths**
- The model integrates market-implied expected losses (and endogenizes loss given default) in a multivariate specification of joint default risk.
- The approach is highly flexible and can be used to quantify an individual institution’s time-varying contribution to systemic solvency risk under normal and stressed conditions.

**Weaknesses**
- Assumptions are required regarding the specification of the option pricing model (for the determination of implied asset and asset volatility of firms).
- Technique is complex and resource intensive.

**Tool**
The Excel VBA-based tool is available with this book.
Contact author: A. Jobst.

Large parts of the material in this chapter are published in the *Sveriges Riksbank Economic Review*, No. 2, pp. 68–106 (Gray and Jobst, 2011b) and IMF Working Paper 13/54 (Jobst and Gray, 2013).
In the wake of the global financial crisis, there has been increased focus on systemic risk as a key aspect of macro-prudential policy and surveillance (MPS) with a view toward enhancing the resilience of the financial sector. MPS is predicated on (1) the assessment of systemwide vulnerabilities and the accurate identification of threats arising from the buildup and unwinding of financial imbalances, (2) shared exposures to macro-financial shocks, and (3) possible contagion/spillover effects from individual institutions and markets owing to direct or indirect connectedness. Thus, it aims to limit, mitigate, or reduce systemic risk, thereby minimizing the incidence and impact of disruptions in the provision of key financial services that can have adverse consequences for the real economy (and broader implications for economic growth). Systemic risk refers to individual or collective financial arrangements—both institutional and market-based—which could either lead directly to systemwide distress in the financial sector or significantly amplify its consequences (with adverse effects on other sectors, in particular capital formation in the real economy). Typically, such distress manifests itself in disruptions to the flow of financial services because of an impairment of all or parts of the financial system that are deemed material to the functioning of the real economy.\footnote{Drehmann and Tarashev (2011) refer to this as a “bottom-up approach,” whereas a “top-down approach” would be predicated on the quantification of expected losses of the system, with and without a particular former, the contribution to systemic risk arises from the initial effect of direct exposures to the failing institution (e.g., defaults on liabilities to counterparties, investors, or other market participants), which could also spill over to previously unrelated institutions and markets as a result of greater uncertainty or the reassessment of financial risk (i.e., changes in risk appetite and/or the market price of risk). Table 26.1 summarizes the distinguishing features of both approaches.}

Although there is still no comprehensive theory of MPS related to the measurement of systemic risk, existing approaches can be distinguished broadly on the basis of their conceptual underpinnings regarding several core principles. There are two general approaches: (1) a particular activity causes a firm to fail, whose importance to the system imposes marginal distress on the system owing to the nature, scope, size, scale, concentration, or connectedness of its activities with other financial institutions (“contribution approach”), or (2) a firm experiences losses from a single (or multiple) large shock(s) because of a significant exposure to the commonly affected sector, country, and/or currency (“participation approach”).\footnote{For a discussion of ways to assess the systemic importance of financial institutions and markets, see IMF, FSB, and BIS (2009).} In the case of the

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1 In a progress report to the G20 (Financial Stability Board, International Monetary Fund, and Bank for International Settlements [FSB, IMF, and BIS], 2011a), which followed an earlier update on macro-prudential policies (FSB/IMF/BIS, 2011b), the FSB takes stock of the development of governance structures that facilitate the identification and monitoring of systemic financial risk as well as the designation and calibration of instruments for macroprudential purposes aimed at limiting systemic risk. The Committee on the Global Financial System (2012) published a report on operationalizing the selection and application of macroprudential policies, which provides guidance on the effectiveness and timing of banking sector–related instruments (affecting the treatment of capital, liquidity, and assets for the purposes of mitigating the cyclical impact of shocks and enhancing systemwide resilience to joint distress events). See Acharya, Cooley, and others (2010) as well as Acharya, Santos, and Yorulmazer (2010) for the implications of systemic risk in MPS in the U.S. context.

2 For a discussion of ways to assess the systemic importance of financial institutions and markets, see IMF, FSB, and BIS (2009).

3 Drehmann and Tarashev (2011) refer to this as a “bottom-up approach,” whereas a “top-down approach” would be predicated on the quantification of expected losses of the system, with and without a particular institution being part of it, which determines the institution’s marginal contribution to systemic risk.
There are several studies on network analysis and agent-based models that are more closely related to the participation approach by modeling how interlinked asset holdings matter in the generation and propagation of systemic risk (Allen, Babus, and Carletti, 2010; Espinosa-Vega and Solé, 2011; Organization for Economic Cooperation and Development, 2012). Haldane and Nelson (2012) underscore this observation by arguing that networks can produce nonlinearity and unpredictability with the attendant extreme (or fat-tailed) events.5

We propose a forward-looking, analytical, market data–based framework for estimating systemic risk in order to fill this gap in the literature. The suggested approach (“Systemic Contingent Claims Analysis” or “Systemic CCA”) extends the risk-adjusted (or economic) balance sheet approach to generate estimates of the joint default risk of multiple institutions. Under this approach, the magnitude of systemic risk depends on the firms’ size and interconnectedness and is defined by the multivariate density of combined expected losses within a given system of financial institutions.

Systemic CCA identifies market-implied linkages affecting joint expected losses during times of stress, which can deliver important insights about the joint tail risk of multiple entities. A sample of firms (as proxy for an entire financial system or parts thereof) is viewed as a portfolio of individual expected losses (with individual risk parameters) whose sensitivity to common risk factors is accounted for by including the statistical dependence of their individual expected losses (“dependence structure”). Like other academic proposals, this approach helps assess individual firms’ contributions to systemic solvency risk (at different levels of statistical confidence). However, by accounting for the time-varying dependence structure, this method links the market-based assessment of each firm’s risk profile with the risk characteristics of other firms that are subject to common changes in market conditions. Based on the expected losses arising from the variation of each individual firm’s expected losses, the joint probability of all firms experiencing distress simultaneously can be estimated.

In addition, the Systemic CCA framework can generate closed-form solutions for market-implied estimates of capital adequacy under various stress test scenarios. By modeling how macroeconomic conditions and bank-specific income and loss elements (net interest income, fee income, trading income, operating expenses, and credit losses) have influenced the changes in market-implied expected losses (as measured by implicit put option values), it is possible to link a particular macroeconomic path to financial sector performance in the future and derive estimates of joint capital needed to maintain current capital adequacy. As a result, the market-implied joint capital need arises from the amount of expected losses relative to current (core) capital levels as well as its escalation.

### TABLE 26.1
General Systemic Risk Measurement Approaches

<table>
<thead>
<tr>
<th>Concept</th>
<th>Contribution approach (“Risk Agitation”)</th>
<th>Participation approach (“Risk Amplification”)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description</td>
<td>A contribution to systemic risk conditional on individual failure due to knock-on effect</td>
<td>Individual resilience to common shock</td>
</tr>
<tr>
<td>Risk transmission</td>
<td>Intra- and intersystem liabilities (“connectedness”)</td>
<td>Expected loss from systemic event due to common exposure and risk concentration</td>
</tr>
<tr>
<td>Risk indicators</td>
<td>Degree of transparency and resolvability (“complexity”)</td>
<td>Market risk exposure (interest rates, credit spreads, currencies)</td>
</tr>
<tr>
<td></td>
<td>Participation in system-critical function/service, e.g., payment and settlement system (“substitutability”)</td>
<td>Economic significance of asset holdings, maturity mismatches debt pressure (“asset liquidation”)</td>
</tr>
<tr>
<td></td>
<td>Avoid/mitigate contagion effect (by containing systemic impact upon failure)</td>
<td>Maintain overall functioning of system and maximize survivorship</td>
</tr>
<tr>
<td></td>
<td>Avoid moral hazard</td>
<td>Preserve mechanisms of collective burden sharing</td>
</tr>
</tbody>
</table>


Note: The policy objectives and different indicators to measure systemic risk under both contribution and participation approaches are not exclusive to each concept. Moreover, the availability of certain types of balance sheet information and/or market data underpinning the various risk indicators varies between different groups of financial institutions, which requires a certain degree of customization of the measurement approach to the distinct characteristics of a particular group of financial institutions.

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1. A comparison of three measures of institution-level systemic risk exposure (Sedunov, 2012) suggests that CoVaR shows good forecasting power of the within-crisis performance of financial institutions during two systemic crisis periods (Long-Term Capital Management in 1998 and Lehman Brothers in 2008). In contrast, Systemic Expected Shortfall (SES) is based on MES after considering different degrees of firm leverage (Acharya, Pedersen, and others, 2012) and Granger causality does not seem to forecast the performance of firm performance reliably during crises.

2. One empirical example of the network literature is based on a Federal Reserve (Fed) data set, which allowed for the mapping of bilateral exposures of 22 global banks that accessed Fed emergency loans in the period 2008 to 2010 (Battiston and others, 2012). The authors find that size is not a relevant factor to determine systemic importance.
<table>
<thead>
<tr>
<th>Description</th>
<th>Conditional Value at Risk (CoVaR)</th>
<th>Conditional Risk (CoRisk)</th>
<th>Systemic Expected Shortfall</th>
<th>Distress Insurance Premium</th>
<th>Joint Probability of Default</th>
<th>Systemic Contingent Claims Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Systemic risk measure</strong></td>
<td>Value at risk</td>
<td>Value at risk</td>
<td>Expected shortfall</td>
<td>Expected shortfall</td>
<td>Expected shortfall</td>
<td>Conditional probabilities</td>
</tr>
<tr>
<td><strong>Conditionality</strong></td>
<td>Percentile of individual return</td>
<td>Percentile of joint default risk</td>
<td>Threshold of capital adequacy</td>
<td>Threshold of capital adequacy</td>
<td>Percentage threshold of system return</td>
<td>Various (individual or joint expected losses)</td>
</tr>
<tr>
<td><strong>Dimensionality</strong></td>
<td>Multivariate</td>
<td>Bivariate</td>
<td>Bivariate</td>
<td>Bivariate</td>
<td>Bivariate</td>
<td>Multivariate</td>
</tr>
<tr>
<td><strong>Dependence measure</strong></td>
<td>Linear, parametric</td>
<td>Linear, parametric</td>
<td>Parametric</td>
<td>Empirical</td>
<td>Parametric</td>
<td>Nonlinear, nonparametric</td>
</tr>
<tr>
<td><strong>Method</strong></td>
<td>Panel quantile regression</td>
<td>Bivariate quantile regression</td>
<td>Dynamic conditional correlation (DCC GARCH) and Monte Carlo simulation</td>
<td>Empirical sampling and scaling; Gaussian and power law</td>
<td>Dynamic conditional correlation (DCC GARCH) and Monte Carlo simulation</td>
<td>Empirical copula</td>
</tr>
<tr>
<td><strong>Data source</strong></td>
<td>Equity prices and balance sheet information</td>
<td>CDS spreads</td>
<td>Equity prices and balance sheet information</td>
<td>Equity prices and CDS spreads</td>
<td>CDS spreads</td>
<td>Equity prices and balance sheet information</td>
</tr>
<tr>
<td><strong>Data input</strong></td>
<td>Quasi-asset returns</td>
<td>CDS-implied default probabilities</td>
<td>Quasi-asset returns</td>
<td>Quasi-asset returns</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gray and Jobst (2010; and 2011b)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Authors.
during extreme market stresses at a very high statistical confidence level (expressed as “tail risk”).\textsuperscript{6}

Because its measure of systemic risk is derived from the joint distribution of individual risk profiles, Systemic CCA addresses the general identification problem of existing approaches. It does not rely on stylized assumptions affecting asset valuation, such as linearity and normally distributed prediction errors, and it generates point estimates of systemic risk without the need of reestimation for different levels of desired statistical confidence. None of the recently published systemic risk models, such as Adrian and Brunnermeier (2008), Acharya, Pedersen, and others (2009, 2010, 2012), and Huang, Zhou, and Zhu (2009, 2010), apply multivariate density estimation, which allows the determination of the marginal contribution of an individual institution to concurrent changes of both the severity of systemic risk and the dependence structure across any combination of sample institutions for any level of statistical confidence.

The chapter is structured as follows: Section 1 explains the general concept of the Systemic CCA framework and provides a detailed description of how joint default risk is modeled via a portfolio-based approach using CCA. Section 2 introduces various extensions to the Systemic CCA framework, including the estimation of contingent claims to the financial sector and the assessment of capital adequacy using market-implied measures of systemwide solvency in the context of macroprudential stress testing. Section 3 presents the estimation results of the Systemic CCA framework for the U.K. banking sector from the IMF’s 2011 Financial Sector Assessment Program (FSAP). The chapter concludes with possible extensions to the presented methodology (and its scope of application) and provides some caveats on systemic risk measurement.

1. METHODOLOGY

A. Overview

Systemic CCA models joint solvency risk by combining the multivariate extension to CCA with the concept of extreme value theory (EVT).\textsuperscript{7} The magnitude of systemic risk jointly posed by multiple firms falling into distress is modeled as a portfolio of individual expected losses (with individual risk parameters) calculated from equity market and balance sheet information. More specifically, these expected losses and the dependence between them are combined to generate a multivariate distribution that formally captures the potential of extreme realizations of joint expected losses as a conditional tail expectation (CTE) as a result of systemwide effects on solvency conditions.

The model comprises two sequential estimation steps in order to measure joint market-implied expected losses. First, each firm’s expected losses (and associated change in existing capital levels) are estimated using an enhanced form of CCA, which has been applied widely to measure and evaluate credit risk at financial institutions.\textsuperscript{8} Second, these expected losses are assumed to follow a generalized extreme value (GEV) distribution, which are combined with a multivariate solution by utilizing a novel application of a nonparametric dependence measure in order to derive the amount of joint expected losses (and changes in corresponding capital levels).

In order to understand individual risk exposures, first, CCA is applied to construct risk-adjusted (economic) balance sheets of financial institutions and estimate their expected losses. In its basic concept, CCA quantifies default risk on the assumption that owners of corporate equity in leveraged firms hold a call option on the firm value after outstanding liabilities have been paid off. The pricing of these state-contingent contracts is predicated on the risk-adjusted valuation of the balance sheet of firms whose assets are stochastic and may be above or below promised payments on debt over a specified period of time. When there is a chance of default, the repayment of debt is considered “risky”—to the extent that it is not guaranteed in the event of default—and thus generates expected losses conditional on the probability and the degree to which the future asset value could drop below the contemporaneous debt payment. Higher uncertainty about changes in future asset value, relative to the default barrier, increases default risk, which occurs when assets decline below the barrier. Thus, the expected loss from default can be valued as an implicit put option.

Thus, the marginal distributions of individual expected losses are combined then with their time-varying dependence structure to generate the multivariate distribution of joint expected losses of all sample firms. This approach of analyzing extreme linkages of multiple entities links the univariate marginal distributions in a way that formally captures both linear and nonlinear dependence and its impact on joint tail risk behavior over time. Given that the simple summation of implicit put option values would presuppose perfect correlation, that is, a coincidence of defaults, the correct estimation of aggregate risk requires knowledge about the dependence structure of individual balance sheets and associated expected

\textsuperscript{6} See also Gray and Jobst (2011a, 2011b, 2011c) for a more general application of this approach to the integrated balance sheets of an entire economy. For first versions of the model, see Gray and Jobst (2009a, 2009b, 2010); Gray and Malone (2010); as well as González-Hermosillo and others (2009). An application of the Systemic CCA approach in the context of Financial Sector Assessment Program stress tests and spillover analysis are published in IMF (2010, 2010a, 2011c, 2011d, 2011e, 2011f, 2012a, 2012b). For an earlier application of the methodology underpinning this framework, see also IMF (2009a, 2009b).

\textsuperscript{7} EVT is a useful statistical concept to study the tail behavior of heavily skewed data, which specifies residual risk at high percentile levels through a generalized parametric estimation of order statistics.

\textsuperscript{8} The CCA is a generalization of option pricing theory pioneered by Black and Scholes (1973) and Merton (1973, 1974). It is based on three principles that are applied in this chapter: (1) the values of liabilities are derived from assets; (2) assets follow a stochastic process; and (3) liabilities have different priorities (senior and junior claims). Equity can be modeled as an implicit call option, while risky debt can be modeled as the default-free value of debt less an implicit put option that captures expected losses. In the Systemic CCA model, advance option pricing is applied to account for biases in the Black-Scholes-Merton specification.
The expected losses of each firm are assumed to be “fat-tailed” and fall within the domain of the GEV distribution, which identifies asymptotic tail behavior of normalized extremes. The joint distribution is estimated iteratively over a prespecified time window with the frequency of updating determined by the periodicity of available data. The specification of a closed-form solution can then be used to (1) determine point estimates of joint expected losses as a multivariate CTE and (2) quantify the contribution of specific institutions to the dynamics of systemic risk.

Systemic CCA accounts for changes in firm-specific and common factors determining individual default risk, their implications for the market-implied linkages between firms, and the resulting impact on the overall assessment of systemic risk. Thus, it accomplishes the essential goal of measuring the extent to which an institution contributes to systemic risk (in keeping with the contribution approach as shown in Table 26.1). The systemic dimension of the model is captured by three properties:

1. **Drawing on the market’s evaluation of a firm’s risk profile.** Given that the framework combines market prices and balance sheet information to inform a risk-adjusted measure of individual default risk, the evaluation of solvency conditions is inherently linked to investor perception as implied by the institution’s equity and equity options (which determine the implied asset value of the firm conditional on its leverage and debt level).

2. **Controlling for common factors affecting the firm’s solvency.** The implied asset value of a firm (which underpins the estimation of implied expected losses) is modeled as being sensitive to the same markets as the implied asset value of every other institution but by varying degrees because of a particular capital structure (and its implications for the market’s perception). Changes in market conditions (and their impact on the perceived risk profile of each firm via its equity price and volatility) establish market-induced linkages among sample firms. Thus, measuring the joint expected losses via option prices links institutions implicitly to the markets in which they obtain equity capital and funding.

3. **Quantifying the chance of simultaneous default via joint probability distributions.** The probability that multiple firms experience a realization of expected losses simultaneously is made explicit by computing their joint probability distributions (which also account for differences in the magnitude of individual expected losses). Hence, the default risk—that is, the likelihood that the implied asset value (including available cash flows from operations and asset sales) falls below the amount of required funding to satisfy debt payment obligations—is assessed not only for individual institutions but for all firms within a system in order to generate estimates of systemic risk.

### B. Model specification and estimation steps

The Systemic CCA framework follows a two-step estimation process. First, the changes to default risk causing a bank to fail (i.e., future debt payments exceeding the asset value of the firm) are modeled as a put option in order to estimate the market-implied expected losses over a certain time horizon (consistent with the application of CCA). Second, these individually estimated expected losses are combined into a multivariate distribution that determines the probabilistic measure of joint expected losses at a systemwide level for a certain level of statistical confidence. Finally, the sensitivity of systemic solvency risk to changes in individual default risk of a single firm (and its implications for the dependence structure of expected losses across all firms) quantifies the degree to which a particular firm contributes to total default risk.

#### Calculating Expected Losses from CCA Using Option Pricing

CCA is a generalization of the option pricing theory pioneered by Black and Scholes (1973) as well as by Merton (1973, 1974) to the corporate capital structure context. When applied to the analysis and measurement of credit risk, it is commonly called the Black-Scholes-Merton (BSoM) model. In its basic concept, it quantifies default risk on the assumption that owners of equity in leveraged firms hold a call option on the firm value after outstanding liabilities have been paid off. The concept of a risk-adjusted balance sheet is instrumental in understanding default risk. The total market value of firm assets, $A$, at any time, $t$, is equal to the sum of its equity market value, $E$, and its risky debt, $D$, maturing at time $T$. The asset value follows a random, continuous process and may fall below the bankruptcy level (“default threshold” or “distress barrier”), which is defined as the present value of promised payments on risky debt $B$, discounted at the risk-free rate. The variability of the future asset value relative to promised debt payments is the driver of credit and default risk. Default happens when assets are insufficient to meet the amount of debt owed to creditors at maturity. Thus, the market-implied

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9 The choice of the empirical distribution function of the underlying data to model the marginal distributions avoids problems associated with using specific parameters that may or may not fit these distributions well (Gray and Jobst, 2009a, 2009b).

10 See also Tarashev, Borio, and Tsatsaronis (2009).
expected losses associated with outstanding liabilities can be valued as an implicit put option (and its cost reflected in a credit spread above the risk-free rate that compensates investors for holding risky debt). The put option value is influenced by the duration of the total debt claim, the leverage of the firm, and the volatility of its asset value.

In the traditional definition of bank balance sheets, a change in accounting assets entails a one-for-one change in book equity. Assets comprise cash and cash equivalents, investments, loans, mortgages, and other cash claims on counterparties as well as noncash claims (derivatives and contingent assets), and liabilities consist of book equity and book value of debt and deposits. When assets change, the full change affects book equity (see Table 26.3). In this context, expected loss is calculated as the probability of default (PD) times a loss-given-default (LGD) times the exposure at default. The expected losses of different exposures are aggregated (using certain assumptions regarding correlation, etc.) and used as an input into loss distribution calculations, which are in turn used for the estimation of risk-weighted assets (RWAs) and the assessment of regulatory capital adequacy.

CCA constructs a risk-adjusted (economic) balance sheet by transforming accounting identities into exposures so that changes in the value of assets are linked directly to changes in the market value of equity and expected losses in an integrated framework. A decline in the value of assets increases expected losses to creditors and leads to less than one-to-one decline in the market value of equity; the amount of change in equity depends on the severity of financial distress (i.e., the perceived impact of anticipated operating—or accounting—losses on equity returns), the degree of leverage, and the volatility of asset returns over time. The underlying “exposure” is represented by the default-free value of the bank’s total debt and deposits after accounting for the (now) higher probability of assets creating insufficient cash flows to meet debt payments (without compromising the integrity of the deposit base) (see Table 26.4). Thus, the integrated modeling of default risk via CCA can quantify the impact on bank borrowing costs of higher (or lower) levels of equity and the market-implied capital assessment. A lower market value of equity would increase the probability of a bank to generate capital losses, which is directly related to higher funding costs.

<table>
<thead>
<tr>
<th>TABLE 26.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional Accounting Bank Balance Sheet</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accounting assets (i.e., cash, reserves, loans, credits, and other exposures)</td>
<td>Debt and deposits (Book equity)</td>
</tr>
</tbody>
</table>

Source: Authors.

First, the amount of expected loss is modeled as a put option based on the individual risk-adjusted balance sheet. The expected loss can be quantified then by viewing default risk as if it were a put option written on the amount of outstanding liabilities, where the present value of debt (i.e., the default barrier) represents the “strike price,” with the value and volatility of assets determined by changes in the equity and equity option prices of the firm. The value of the put option increases the higher the probability of the implied asset value falls below the default barrier over a predefined horizon. Such probability is influenced by changes in the level and the volatility of their implied asset value reflected in the institution’s equity and equity option prices conditional on its capital structure. Thus, the present value of market-implied expected losses associated with outstanding liabilities in keeping with the traditional BSM model can be valued as an implicit (European) put option value:

\[
P_e(t) = Be^{-(r-T)t} \Phi \left( -d - \sigma \sqrt{T-t} \right) - A(t) \Phi(-d), \tag{26.1}\]

with the present value of debt \(B\) as strike price on the asset value \(A(t)\) and asset return volatility:

\[
\sigma_A = \frac{E(t) \sigma_E}{A(t) \Phi(d)} \left( \frac{1 - Be^{-(r-T)t} \Phi \left( d - \sigma A \sqrt{T-t} \right)} {A(t) \Phi(d)} \right) \sigma_A, \tag{26.2} \]

over time horizon \(T-t\), where \(\sigma_E\) is the (observable) equity volatility, \(E(t)\) is the equity price, \(r\) is the risk-free discount rate, and \(\Phi(d)\) is the cumulative probability of the standard normal density function of the leverage:

\[
d = \ln \left( \frac{A(t)}{B} \right) + \left( r + \frac{\sigma_E^2}{2} \right) (T-t) / \sigma_A \sqrt{T-t}, \tag{26.3} \]

changes in \(A(t)\) relative to outstanding debt claims. This specification of option price-based expected losses, however, does not incorporate skewness, kurtosis, and stochastic volatility, which can account for implied volatility smiles of equity prices. These shortcomings of the conventional Merton model can be addressed with more advanced valuation models, such as the Gram-Charlier expansion of the density function of asset changes (Backus, Foresi, and Wu, 2004) or a jump diffusion process of asset value changes in order to achieve...

<table>
<thead>
<tr>
<th>TABLE 26.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-Adjusted (CCA) Bank Balance Sheet</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Implied) market value of assets (A) (i.e., cash, reserves, and implied market value of “risky” assets)</td>
<td>“Risky” debt (D) (= default-free value of debt and deposits minus expected losses to bank creditors)</td>
</tr>
<tr>
<td>(Observable) market value of equity (E) (= market capitalization)</td>
<td></td>
</tr>
</tbody>
</table>

Source: Authors.

Note: CCA = contingent claims analysis.
more robust and reliable estimation results. Some of these approaches have been applied in the empirical use of the Systemic CCA framework in this chapter.

Given that the asset value is influenced by the empirical irregularities contained in the Merton model (which also affects the model-based calibration of implied asset volatility), it is estimated directly from observable equity option prices. The state-price density (SPD) of the implied asset value is estimated from the risk-neutral probability distribution of the underlying asset price at the maturity date of equity options with different strike prices, without any assumptions on the underlying asset diffusion process (which is assumed to be log-normal in the Merton model). The implied asset value is defined as the expectation over the empirical SPD by adapting the well-established Breeden and Litzenberger (1978) method, together with a semiparametric specification of the Black-Scholes option pricing formula (Aït-Sahalia and Lo, 1998). More specifically, this approach uses the second derivative of the call pricing function (on European options) with respect to the strike price (rather than option price). Estimates are based on option contracts with identical time to maturity, assuming a continuum of strike prices. The SPD of the equity price includes the risk-neutral expectation of the implied asset value and removes the impact of differences of higher moments of both the equity price and the implied asset value dynamics over the chosen time horizon. Thus, the current implied asset value can be transposed from the SPD of the equity price conditional on the contemporaneous leverage of the firm.

Note, however, that the presented valuation model is subject to varying degrees of estimation uncertainty and parametric assumptions. The option pricing model (given its specific distributional assumptions, the derivation of both implied asset volatility, and assumptions about the default barrier) could fail to capture some relevant economics that are needed to fully understand default risk and thus could generate biased estimators of expected losses. Moreover, equity prices might not only reflect fundamental values because of both shareholder dilution and trading behavior that obfuscate proper economic interpretation.

**Estimating the Joint Expected Losses from Default Risk**

Second, the individually estimated expected losses are combined to determine the magnitude of default risk on a systemwide level. The distribution of expected losses is assumed to be “fat-tailed” in keeping with EVT. This specification of individual loss estimates then informs the estimation of the dependence function. As part of a three-step subprocess, we define the univariate marginal density functions of all firms, which are then combined with their dependence function in order to generate an aggregate measure of default risk. This multivariate setup underpinning the Systemic CCA framework formally captures the realizations of joint expected losses. We can then use tail risk estimates, such as the conditional VaR (or expected shortfall, or ES), in order to gauge systemic solvency risk in times of stress.

1. Estimating the Marginal Distributions of Individual Expected Losses. We first specify the statistical distribution of individual expected losses (based on the series of put option values obtained for each firm) in accordance with EVT. This is a general statistical concept of deriving a limit law for sample maxima. Let the vector-valued series

\[ X_m^n = P_{i,m}(t), \ldots, P_{n,m}(t) \]  

(26.4)

denote independent, identically distributed random observations of expected losses (i.e., a total of \( n \) number of daily put option values \( P_{i,m}(r) \) up to time \( t \)), each estimated according to equation (26.1) over a rolling window of \( r \) observations with periodic updating (e.g., a daily sliding window of 120 days) for \( j \in m \) firms in the sample. The individual asymptotic tail behavior is modeled in accordance with the Fisher-Tippett-Gnedenko theorem (Fisher and Tippett, 1928; Gnedenko, 1943), which defines the attribution of a given distribution of normalized maxima (or minima) to be of extremal type (assuming that the underlying function is continuous on a closed interval). Given

\[ X^n = \max(P_{1,m}(t), \ldots, P_{n,m}(t)), \]  

(26.5)

evenly there exists a choice of normalizing constants \( \beta^n > 0 \) and \( \alpha^n \), such that the probability of each ordered \( n \)-sequence of normalized sample maxima \( (X^n - \alpha^n)/\beta^n \) converges to the nondegenerate limit distribution \( H(x) \) as \( n \to \infty \) and \( x \in \mathbb{R} \), so that

---

12 The suggested approach takes into account the nonlinear characteristics of asset price changes, which deviate from normality and are frequently influenced by stochastic volatility. The BSM model has been shown to consistently understate spreads (Jones, Mason, and Rosenfeld, 1984; Ogden, 1987), with more recent studies pointing to considerable pricing errors because of its simplistic nature.

13 Other extensions are aimed at imposing more realistic assumptions, such as the introduction of stationary leverage ratios (Collin-Dufresne and Goldstein, 2001) and stochastic interest rates (Longstaff and Schwartz, 1995). Incorporating early default (Black and Cox, 1976) does not represent a useful extension in this context given the short estimation and forecasting time window.

14 Given that the implied asset value is derived separately, this approach avoids the (problematic) “two-equations-two-unknowns” approach to derive implied assets and asset volatility based on Jones, Mason, and Rosenfeld (1984), which was subsequently extended by Ronn and Verma (1986) to a single equation to solve two simultaneous equations for asset value and volatility as two unknowns. Duan (1994) shows that the volatility relation between implied assets and equity could become redundant if the equity volatility is stochastic. An alternative estimation technique for asset volatility introduces a maximum likelihood approach (Erickson and Reneby, 2004, 2005).

15 Because available strike prices always are discretely spaced on a finite range around the actual asset value, interpolation of the call pricing function inside this range and extrapolation outside this range are performed by means nonparametric (local polynomial) regression of the implied volatility surface (Rockley, 1997).

16 For instance, during the financial crisis, rapid declines in market capitalization of firms were not only a signal about future solvency risk but also reflected a “flight to quality” motive that was largely unrelated to expectations about future firm earnings or profitability.
falls within the maximum domain of attraction (MDA) of the GEV as limiting distribution of maxima of dependent random variables (Coles, Heffernan, and Tawn, 1999; Poon, Rockinger, and Tawn, 2003; Jobst, 2007) with the cumulative distribution function

\[ F_{\beta \gamma}(x) = \lim_{n \to \infty} \Pr \left( \frac{X_i - \beta \gamma}{\beta} \leq y \right) = \left[ F(\beta \gamma + \alpha) \right]^n \rightarrow H(x) \]  

(26.6)

and differencing equation (26.7) above as \( H_{\mu, \sigma, \xi}(x) = \frac{d}{dx} H_{\mu, \sigma, \xi}(x) \) yields the probability density function,

\[ h_{\mu, \sigma, \xi}(x) = \frac{1}{\sigma} \left( 1 + \frac{x - \mu}{\sigma} \right)^{-\xi} \times \exp \left( -\left( 1 + \frac{x - \mu}{\sigma} \right)^{-\xi} \right) \]  

(26.8)

Thus, the \( j \)th univariate marginal density function of each expected loss series converging to GEV in the limit is defined as

\[ y_j(x) = \left( 1 + \frac{x - \mu_j}{\sigma_j} \right)^{-\xi_j} \]  

(26.9)

where \( 1 + \frac{x - \mu}{\sigma} > 0 \), scale parameter \( \sigma > 0 \), location parameter \( \mu_j \in \mathbb{R} \), and shape parameter \( \xi_j \neq 0 \). These moments are estimated concurrently by means of numerical iteration via maximum likelihood (ML), which identifies possible limiting laws of asymptotic tail behavior, that is, the likelihood of even larger extremes as the level of statistical confidence approaches certainty. The ML estimator in the GEV model is evaluated numerically by using an iteration procedure (e.g., over a rolling window of \( t \) observations with periodic updating) to maximize the likelihood \( \prod_{i=1}^{n} h_{\mu, \sigma, \xi}(x_i) | \theta \) over all three parameters \( \theta = (\mu, \sigma, \xi) \) simultaneously, where the linear combinations of ratios of spacings estimator serves as an initial value (Jobst, 2014).\(^{18}\)

2. Estimating the Dependence Structure of Individual Expected Losses. Second, we define a nonparametric, multivariate dependence function between the marginal distributions of expected losses by expanding the bivariate logistic method proposed by Pickands (1981) to the multivariate case and adjusting the margins according to Hall and Tajvidi (2000) so that

\[ \Upsilon(\omega) = \min \left\{ 1, \max \left\{ n \sum_{i=1}^{n} \frac{y_i / y_n}{\omega}, \omega, 1 - \omega \right\} \right\} \]  

(26.10)

where \( y_j = \sum_{i=1}^{n} y_i / n \) reflects the average marginal density of all \( i \) in \( n \) put option values and \( 0 \leq \omega \leq \max (\omega_1, \ldots, \omega_m) \leq \Upsilon(\omega) \leq 1 \) for all \( 0 \leq \omega \leq 1 \). \( \Upsilon(\cdot) \) represents a convex function on \([0,1]\) with \( \Upsilon(0) = \Upsilon(1) = 1 \), that is, the upper and lower limits of \( \Upsilon(\cdot) \) are obtained under complete dependence and mutual independence, respectively. It is estimated iteratively (over a rolling window of \( t \) observations with periodic updating at a frequency that is consistent with that in equation (26.5), subject to the optimization of the \((m-1)\)-dimensional unit simplex:

\[ S_m = \left\{ (\omega_1, \ldots, \omega_m) | \omega_j \geq 0, 1 \leq j \leq m-1; \sum_{j=1}^{m-1} \omega_j \leq 1 \text{ and } \omega_m = 1 - \sum_{j=1}^{m-1} \omega_j \right\} \]  

(26.11)

which establishes the degree of coincidence of multiple series of cross-classified random variables similar to a chi-statistic that measures the statistical likelihood that observed values differ from their expected distribution. This specification stands in contrast to a general copula function that links the marginal distributions using only a single (and time-invariant) dependence parameter.

3. Estimating the Joint Distribution and a Tail Risk Measure of Joint Expected Losses. We then combine the marginal distributions of these individual expected losses with their dependence structure to generate a multivariate extreme value distribution over the same estimation period as earlier.\(^{20}\) The resultant cumulative distribution function is specified as

\[ G_{m,n}(x) = \exp \left\{ -\left( \sum_{i=1}^{n} y_i \right) \Upsilon(\omega) \right\} \]  

(26.12)

\(^{17}\) The upper tails of most (conventional) limit distributions (weakly) converge to this parametric specification of asymptotic behavior, irrespective of the original distribution of observed maxima (unlike parametric VaR models).

\(^{18}\) Note that the ML estimator fails for \( \xi < -1 \) since the likelihood function does not have a global maximum in this case. However, a local maximum close to the initial value can be attained.

\(^{19}\) Note that the marginal density of a given extreme relative to the average marginal density of all extremes is minimized \((\leq 1)\) across all firms \( j \in m \), subject to the choice of factor \( \omega \). A bivariate version of this approach has been implemented in Jobst and Kamil (2008).

\(^{20}\) The analysis of dependence in this presented approach is completed separately from the analysis of marginal distributions and thus differs from the classical approach, where multivariate analysis is performed jointly for marginal distributions and their dependence structure by considering the complete variance–covariance matrix, using techniques like the Multivariate Generalized AutoRegressive Conditional Heteroskedasticity approach. To obtain a multivariate distribution function, the dependence function combines several marginal distribution functions in accordance with Sklar’s (1959) theorem on constructing joint distributions with arbitrary marginal distributions via copula functions (which completely describes the dependence structure and contains all the information to link the marginal distributions). For the formal treatment of copulas and their properties, see Hutchinson and Lai (1990); Dall’Aglio, Kotz, and Salinetti (1991); and Joe (1997).
with corresponding probability density function
\[
g_{t,m}(x) = \frac{1}{\hat{\sigma}_{t,m}} \left( \sum_{j=1}^{m} Y_{t,i} \right)^{-1} \exp \left\{ -\left( \sum_{j=1}^{m} Y_{t,i} \right) Y_{t,i} \right\},
\]
(26.13)
at time \( t = t+1 \) by maximizing the likelihood \( \prod_{j=1}^{m} g_{t,m}(x | \theta) \) over all three parameters \( \theta = (\mu, \alpha, \xi) \) simultaneously so that the ML estimate of the true values \( \hat{\theta}_0 \) is
\[
\hat{\theta}_0 = \arg \max_{\theta} \hat{\gamma}(\theta | x) \rightarrow \hat{\theta}_0.
\]
(26.14)
Finally, we obtain the joint ES (or conditional VaR) as a measure of CTE. ES defines the probability-weighted residual measure of CTE. VaR is defined as
\[
\text{VaR}_{t,m,a} = \sup \left\{ G_{t,m,a}(a) \left| Pr[z_{t,i} > G_{t,m,a}(a)] \geq a = 0.95 \right. \right\},
\]
(26.16)
with the point estimate of joint potential losses of \( m \) firms at time \( t \) defined as
\[
G_{t,m,a}(a) = \mu_{t,m} + \hat{\sigma}_{t,m} \xi_{t,m} \left( -\frac{\ln(a)}{Y_t(a)} \right)^{-\frac{1}{\xi_{t,m}}} - 1.
\]
(26.17)
The contribution of each firm can be determined by calculating the cross-partial derivative of the joint distribution of expected losses. The joint ES also can be written as a linear combination of individual ES values, \( ES_{t,m,a} \), where the relative weights \( \psi_{t,m,a} \) (in the weighted sum) are given by the second order cross-partial derivative of the inverse of the joint probability density function \( G_{t,m,a}^{-1}(\cdot) \) to changes in both the dependence function \( Y_t(\cdot) \) and the individual marginal severity \( Y_{t,i} \) of expected losses. Thus, the contribution can be derived as the partial derivative of the multivariate density to changes in the relative weight of the univariate marginal distribution of expected losses and its impact on the dependence function (of all expected losses of sample firms) at the specified percentile. By rewriting \( ES_{t,m,a} \) in equation (26.15), we obtain the following:
\[
ES_{t,m,a} = \sum_{j} \psi_{t,m,a} \text{E} \left[ z_{t,j} \right] \left| z_{t,m} \geq G_{t,m,a}(a) = \text{VaR}_{t,m,a} \right].
\]
(26.18)

where the relative weight of institution \( j \) is defined as the marginal contribution,
\[
\psi_{t,m,a} = \frac{\partial G_{t,m,a}(a)}{\partial Y_t(Y_t(a))}
\]
(26.19)
s.t. \( \sum_{j} \psi_{t,j,a} z_{t,j} \leq z_{t,m} \) attributable to the joint effect of both the marginal density and the change of the dependence function owing to the presence of institution \( j \in m \) in the sample.

2. EXTENSIONS OF THE SYSTEMIC CCA FRAMEWORK

A. Integrated market-implied capital assessment using CCA and Systemic CCA

Given that the evaluation of default risk is inherently linked to perceived riskiness as implied by the changes in equity and equity option prices, the risk-adjusted (economic) balance sheet approach—and its multivariate extension in the form of Systemic CCA—provides an integrated analytical framework for a market-based assessment of individual and systemwide solvency. Higher expected losses to creditors lead to less than a one-to-one decline in the market value of equity, depending on the severity of the perceived (and actual) decline in the value of assets, the degree of leverage, and the volatility of assets. Thus, the interaction between these constituent elements of the risk-adjusted (economic) balance sheet gives rise to a market-based capital assessment, which links changes in the value of assets directly to the market value of equity.

As opposed to the regulatory definition of capitalization (based on the discrete nature of accounting identities), the CCA-based capital assessment hinges on the historical dynamics of implied asset values and their effect on the magnitude of expected losses. Two broad indicators serve to illustrate this point: (1) the market-implied capital adequacy ratio (MCAR), which is defined as the ratio between the amount of market capitalization divided by the implied asset value of a firm; and (2) the expected loss ratio (“EL ratio”) between individual expected losses (at a defined percentile of statistical confidence) and the amount of market capitalization (see Figure 26.1, first panel). Both ratios also can be generated on a systemwide basis, with inputs defined as sample aggregates of input variables to the multivariate specification of the Systemic CCA. In this case, the EL ratio is defined by magnitude of joint expected losses (at the 95th percentile, for instance) relative to aggregate market capitalization of all sample firms (see Figure 26.1, second panel).

The market-implied capital assessment also can help identify a capital shortfall. The MCAR represents an important analogue to the regulatory definition of capital adequacy, which reflects the market’s perception of solvency (based on the implied value of assets relative to the prevailing default risk) and is completely removed from prudential determinants of default risk affecting a bank’s capital assessment,
Andreas A. Jobst and Dale F. Gray

Figure 26.1 Integrated Market-Based Capital Assessment Using CCA and Systemic CCA (based on nonlinear relation between CCA-based CAR, EL ratio, fair value credit spread, and implied asset volatility)

Source: Authors. Note: CAR = capital adequacy ratio; CCA = contingent claims analysis; EL = expected losses; MCAR = market-implied CAR.
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such as risk-weighted assets. In the empirical section of the chapter, the relation between the MCAR and regulatory capital adequacy is investigated further, which establishes the possibility of defining a systematically based capital shortfall using CCA-based measures of solvency (see Section 3).

**B. Estimating the contingent liabilities to the public sector**

Large implicit government financial guarantees (valued as contingent liabilities) create significant valuation linkages between sovereigns and financial sector risks. These linkages could give rise to a destabilization process that increases the susceptibility of public finances to the potential impact of distress in an outsized financial sector (see Figure 26.2). The financial crisis that started in 2007 is a stark reminder of how public sector support measures provided to large financial institutions can result in considerable risk transfer to the government, which places even greater emphasis on long-term fiscal sustainability. A negative feedback loop between the financial sector risks and fiscal policy arises because a decline in the market value of sovereign debt not only weakens the implicit sovereign guarantees to systemically important financial institutions (and large banks in particular) but also raises default risk in the financial sector overall (which increases the cost of borrowing). There is also a higher likelihood of downgrades to financial institutions following downgrades of sovereign credit ratings, which establish an upper ceiling to their unsecured funding ratings. This linkage between the creditworthiness of the sovereign and the financial system (in particular, the banking sector) is prone to perpetuate a negative feedback effect that is accentuated by the lender–borrower channel in situations where financial institutions hold large exposures to public sector entities.

Based on CCA, the market-implied expected losses calculated from equity market and balance sheet information of financial institutions can be combined with information from their credit default swap (CDS) contracts to estimate the government’s contingent liabilities. Because systemically important institutions enjoy an implicit government guarantee, their creditors have an expectation of public sector support in the

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22 See Gray and Jobst (2009b). See also Gapen (2009) for an application on CCA to the measurement of contingent liabilities from government-sponsored enterprises in the United States.
case of failure.\textsuperscript{25} Conversely, the cost of insuring the nonguaranteed debt (as reflected in the senior CDS spread) should capture only the expected loss retained by such financial institutions (and borne by unsecured senior creditors), after accounting for any implicit public sector support that is due to their systemic significance. Thus, the market-implied government guarantee for large banks (as prime candidates for the status of systemically important financial institutions) can be derived heuristically as the difference between the total expected loss (i.e., the value of a put option \( P_E(t) \) derived from the bank’s equity price) and the value of an implicit put option \( P_{CDS}(t) \) derived from the bank’s CDS spread—assuming that equity values remain unaffected (which is very likely when banks are close to the default barrier, as the call option value from the residual claim of equity to future profits after debt payment converges to zero).

By replacing individual expected losses with this measure of contingent liabilities, the Systemic CCA framework can be used to derive an estimate of systemic risk from joint contingent liabilities. Given that the put option value \( P_{CDS}(t) \leq P_E(t) \), it reflects the expected losses associated with default net of any financial guarantees, that is, residual default risk on unsecured senior debt. \( P_{CDS}(t) \) can be written as

\[
P_{CDS}(t) = \left(1 - \exp\left(-\frac{s_{CDS}(t)}{10,000}\right)\right) \times \left(\frac{B}{D(t)} - 1\right) \left(T - t\right) \times 10,000, \quad (26.20)
\]

after rearranging the specification of the CDS spread (in basis points), \( s_{CDS}(t) \), under the risk-neutral measure as

\[
s_{CDS}(t) = -\frac{1}{T-t} \ln\left(1 - \frac{P_{CDS}(t)}{Be^{-r(T-t)}}\right) \times \left(\frac{B}{D(t)} - 1\right), \quad (26.21)
\]

assuming a default probability,

\[
1 - \exp\left(-\frac{s_{CDS}(t)}{10,000}\right) = 1 - \exp(-\varphi t) = \frac{P_{CDS}(t)}{Be^{-r(T-t)}}, \quad (26.22)
\]

at time \( t \), constant hazard rate \( s_{CDS}(t) = \varphi \), and the implied yield to maturity defined by

\[
D(t) = Be^{-r(T-t)} \iff e^{-r(T-t)} = \frac{D(t)}{B} = \frac{Be^{-r(T-t)} - P_E(t)}{B}, \quad (26.23)
\]

so that \((1 - s_{CDS}(t))Be^{-r(T-t)} = PD \times LGD\) over one period \( T-t=1 \). A linear adjustment of \( BID(t) - 1 \) is needed in order to control for the impact of any difference between the market price and the repayable face value of outstanding debt on the fair pricing of credit protection. If outstanding debt trades above par, \( D(t) > B \), the recovery of the nominal value under the CDS contract ("recovery at face value") drops below the recovery rate implied by the market price of debt ("recovery at market value"). As a result, the CDS spread, \( s_{CDS}(t) \), drops below the bond spread.\textsuperscript{24} Conversely, a below-par bond price pushes up the implied recovery rate of CDS, causing a markup of CDS spreads over comparable bond spreads for the same estimated default risk.

Estimated expected losses derived from both equity prices and credit spreads can be combined to derive the market-implied degree to which potential public sector support depresses the cost of credit. Given \( P_{CDS}(t) \), the ratio

\[
\alpha(t) = 1 - \frac{P_{CDS}(t)}{P_E(t)}, \quad (26.24)
\]

defines the share of expected loss covered by implicit (or explicit) government guarantees that depress the CDS spread below the level that would be warranted by the equity-implied default risk.\textsuperscript{25} The following equation,

\[
\alpha(t)P_E(t) = \left(1 - \frac{P_{CDS}(t)}{P_E(t)}\right) P_E(t) = P_E(t) - P_{CDS}(t), \quad (26.25)
\]

represents the fraction of default risk covered by the government (and no longer reflected in the market-implied default risk), and \((1 - \alpha(t))P_E(t)\) is the risk retained by an institution and reflected in the senior CDS spread. Thus, the time pattern of the government’s contingent liabilities then can be used to replace the marginal densities of expected losses in equation (26.9) in order to derive the systemic risk from the implicit public sector support to the banking sector according to the specification in equation (26.12).

Although this definition of market-implied contingent liabilities provides a useful indication of possible sovereign risk transfer, the presented estimation method depends on a variety of assumptions that influence the assessment of the likelihood of public sector support, especially at times of extreme stress. The extent to which the equity put option values differ from the one implied by CDS spreads might reflect distortions stemming from the modeling choice (and the breakdown of efficient asset pricing in situations of illiquidity), changes in market conditions, and the capital structure impact of government interventions, such as equity dilution in the wake of capital injections, beyond the influence of explicit or implicit guarantees.

Even though the equality condition of default probabilities derived from equity prices and CDS spreads implies that positive \( \alpha(t) \) values cannot exist in absence of arbitrage, empirical

\textsuperscript{24} The difference in recovery values (often referred to as “basis”) implied by a divergence of CDS and bond spreads is approximated by using the fair value CDS (FVCDS) spread and the fair value option adjusted spread (FVOAS) reported by Moody’s KMV (MKMV) CreditEdge. Both FVOAS and FVCDS represent credit spreads (in basis points) for the bond and CDS contract of a particular firm based on risk horizon of \( t \) years (where \( t = 1 \)–10 years). Both spreads imply an LGD determined by the industry category. In practice, this adjustment factor is very close to unity for most of the cases, with a few cases where the factor falls within a 20 percentage point range (0.9 to 1.1).

\textsuperscript{25} Note that the estimation assumes a European put option, which does not recognize the possibility of premature execution. This might overstate the actual expected losses inferred from put option values in comparison with the put option derived from CDS spreads.
evidence during times of stress suggest otherwise.26 Carr and Wu (2007) show that for many firms the put option values from equity options and CDSs are, indeed, closely related.27 In stress situations, however, the implicit put options from equity markets and CDS spreads can differ in their capital structure impact and thus should be priced differently. Besides guarantees, there are several distortions that could set apart put option values derived from CDS and equity prices, even if the risk-neutral default probability (RNDP) implied by the CDS spread (based on an exponential hazard rate) were the same as the RNDP component of the equity put option value. Some of these factors include (1) the recovery-at-face-value assumption underlying CDS spreads, and (2) different risk horizons of put option values derived from CDS and equity prices. We address these two potential sources of distortion via an adjustment for “basis risk” in equation (26.20).

C. Systemic CCA and stress testing

The Systemic CCA framework also can be used to project the dynamics of systemic solvency risk for the purposes of assessing capital adequacy under stress scenarios. Based on the historical distribution of firm-specific market-implied expected losses (which can be generated using conventional or more advanced option pricing techniques, future changes in market-implied default risk of each firm can be estimated over the selected forecast horizon (based on their macro-financial linkages under different stress scenarios) and finally combined to define systemwide solvency risk within the Systemic CCA framework.

First, a suitable univariate input is defined. Individual CCA-based estimates of market-implied expected losses can generate the following measures that support a firm-specific assessment of capital adequacy (and which have been applied empirically already):

1. capital shortfall based on the amount of expected losses in excess of existing common (core) equity Tier 1 capital above the regulatory minimum,28 and

2. capital shortfall based on the MCAR generated from the change in market capitalization relative to the asset value under the impact of expected losses.

The first measure is presented in the empirical section of this chapter (see Section 3), which illustrates the practical application of the Systemic CCA framework for macroprudential stress testing. The second measure of MCAR is presented in the descriptive part of the section but is not pursued further in the application of the stress test scenarios.

Second, empirical and theoretical (endogenous) models can be used to specify the macro-financial linkages of expected losses. By modeling how the impact of macroeconomic conditions on income components and asset impairment (such as net interest income, fee income, trading income, operating expenses, and credit losses) has influenced a bank’s market-implied expected losses in the past, it is possible to link individual estimates of expected losses (and their implications for potential capital shortfall) to a particular macroeconomic path (and associated financial sector performance) in the future (IMF, 2010, 2011a, 2001b, 2011d, 2012a, 2012b). Alternatively, the historical sensitivity of the main components of the option pricing formula themselves (i.e., the implied asset value and asset volatility) can be determined by applying stress conditions directly to the theoretical specification of market-implied expected losses. Also, the implied asset value underpinning the put option value of each sample bank could be adjusted by the projected profitability under different stress scenarios in order to determine the corresponding change in the level of market-implied expected losses (IMF, 2011c, 2011e).

Third, the joint potential capital shortfall is derived from estimating the multivariate density of each bank’s individual marginal distribution of forecast expected losses (if any) and their dependence structure. If the number of forecast expected losses is insufficient because of the low frequency of macroeconomic variables chosen for the specification of macro-financial linkages (e.g., quarterly values over a five-year forecast horizon), the time series of expected losses could be supplemented by historical expected losses to a point when the number of observations are sufficient for the estimation of joint capital shortfall.

3. EMPIRICAL APPLICATION: STRESS TESTING SYSTEMIC RISK FROM EXPECTED LOSSES IN THE U.K. BANKING SECTOR

This section summarizes the findings from applying the Systemic CCA framework as a market-based top-down (TD) solvency stress testing model as part of the IMF’s FSAP stress valuation and, thus, individual estimates of market-implied capital shortfall. In the empirical application of Systemic CCA for capital assessment, Tier 1 capital was used for consistency purposes (because of divergent regulatory treatments of more junior capital tiers).
test of the U.K. banking sector (IMF, 2011e). The model was estimated based on market and balance sheet information of the five largest commercial banks, the largest building society, and the largest foreign retail bank using daily data between January 3, 2005, and end-March 2011. The individual market-implied expected losses were treated as a portfolio, whose combined default risk over a prespecified time horizon generated estimates of individual firms’ contributions to systemic risk in the event of distress (“tail risk”).

A. Illustrating the nonlinearity of historical estimates of individual and joint expected losses

Based on the historical estimates of expected losses, the risk-based valuation of default risk within the Systemic CCA framework supports an integrated market-based capital assessment (see Section 2). Figure 26.3 illustrates the nonlinear relation between the daily estimates of the MCAR, the EL ratio, the fair value credit spread, and the implied asset volatility of one selected sample firm in accordance with the stylized example shown in Figure 26.1. As expected, the rising default risk during the financial crisis establishes a high sensitivity of MCAR to a precipitous increase of expected losses at a level consistent with the regulatory measure of capital adequacy (see Figure 26.3, second panel). However, the growing gap between MCAR and the Core Tier 1 ratio toward the end point of the estimation time period implies that capitalization of the selected bank was perceived by investors as much lower than what regulatory standards would otherwise suggest.

Similarly, the interaction between these variables also can be shown on a systemwide basis (see Figure 26.4). For all firms in the sample, the firm-specific variables underpinning the presented measure were replaced with systemwide measures, such as the joint EL ratio (defined as the sum of 95 percent VaR values of individual expected losses or the joint 95 percent ES divided by the aggregate market capitalization) and the aggregate MCAR. The empirical results suggested an even higher sensitivity of MCAR to changes in systemwide expected losses than in the case of the single-firm example shown in Figure 26.3. In addition, the comparison of MCAR to the regulatory benchmark (Core Tier 1) indicates that the improvement of solvency conditions after the peak of the financial crisis in March 2009 appeared more protracted across all sample firms (see Figure 26.4, bottom chart).

B. Estimating expected losses under stress conditions based on macro-financial linkages

Individual market-implied expected losses under stress conditions were derived from their historical sensitivity to changes in macro-financial conditions and bank performance. Expected losses under different stress scenarios were forecast using two different methods for the specification of the macro-financial linkages of expected losses as measured by the implicit put option values of sample firms (see Section 2.B):

- In the first model (“IMF satellite model”), expected losses were estimated using a dynamic panel regression specification with several macroeconomic variables (short-term interest rate $+\Delta$, long-term interest rate $-\Delta$, real GDP $-\Delta$, and unemployment $+\Delta$) and several bank-specific output variables (net interest income $+\Delta$, operating profit before taxes $-\Delta$, credit losses $+\Delta$, leverage $+\Delta$, and funding gap $+\Delta$).

- In the second model (“structural model”), the value of implied assets of each bank as at end-2010 is adjusted by forecasts of operating profit and credit losses generated in the Bank of England’s Risk Assessment Model for Systemic Institutions (RAMSI) in order to derive a revised put option value (after reestimating implied asset volatility), which determines the market-implied capital loss.

The individually estimated expected losses then were transposed into a capital shortfall. The potential capital loss was determined on an individual basis as the marginal change of expected losses over the forecast horizon (2011-2015) and evaluated for each macro-financial approach and under each of the specified adverse scenarios. As an alternative, the market-implied CAR (see Figures 26.3 and 26.4) could have been used as a basis for assessing capital adequacy and the extent to which expected losses result in potential capital shortfall. However, in the context of this exercise, the CCA-based results were transposed into conventional (regulatory)

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29 The Systemic CCA framework also served as a means of cross-validating the stress testing results from the bottom-up exercise conducted jointly with the Financial Services Authority (FSA).

30 The sample comprised the following firms: Hongkong and Shanghai Banking Corporation (HSBC), Barclays, Royal Bank of Scotland (RBS), Lloyds’s Banking Group, Standard Chartered Bank, Nationwide, and Santander U.K. Key inputs used were the daily implied asset values derived from the expectation over the state-price density (SPD) using equity call option price data over the sample time period with a time to maturity of three months. Besides the maturity tenor, option price data also included information on strike prices as well as actual and corrected spot prices (with implied dividends taken into account). The multivariate distribution of expected losses (i.e., the implicit put option value over a one-year horizon) across all sample firms was estimated over a rolling window of 60 working days (i.e., three months) with daily updating, consistent with the maturity tenor of equity option prices used for the estimation of the implied asset value of each firm.

31 The plus or minus signs indicate whether the selected variable exhibited a positive or negative regression coefficient. To be included in the model, the variables needed to be statistically significant at least at the 10 percent level.

32 This approach also assumed a graduated increase of the default barrier consistent with the transition period to higher capital requirements for the most junior levels of equity under Basel III standards (Basel Committee on Banking Supervision, 2012).
Figure 26.3 United Kingdom: Integrated Individual Market-Based Capital Assessment Based on CCA-Derived Estimates of Expected Losses
Andreas A. Jobst and Dale F. Gray

Figure 26.4 United Kingdom: Integrated Aggregate Market-Based Capital Assessment Based on Systemic CCA-Derived Estimates of Joint Expected Losses

Source: Authors.

Note: CCA = contingent claims analysis; ES = expected shortfall; VaR = value at risk.
measures of solvency, assuming that any increase in expected losses in excess of existing capital buffers constitutes a shortfall of Tier 1 capital (which was chosen as the closest approximation of common equity in CCA).

Finally, individual capital losses then were combined using the Systemic CCA framework in order to determine the joint capital shortfall relative to the current solvency level over the forecast horizon. The univariate density functions of individual capital shortfalls and their dependence structure were combined to a multivariate probability distribution of joint capital shortfall (with a five-year sliding window and monthly updates). The estimated distribution allowed for an assessment of joint capital adequacy under stress conditions at different levels of statistical confidence (e.g., 50th and 95th percentiles) as opposed to the summation of individual institutions’ capital adequacy (see Section 1). Any capital shortfall arose when the level of Tier 1 capital after accounting for joint capital losses fell below the hurdle rate of regulatory capital defined by the transition period in Basel III (and compared against the FSA’s interim capital regime requirements).

Estimation results of the joint capital shortfall suggested that any impact from the realization of systemic solvency risk would have been limited even in a severe recession. The joint capital loss at the 50th percentile would have been contained under all adverse scenarios. Existing capital buffers were sufficient to absorb the realization of central case (median) joint solvency risks. The severe double-dip recession scenario had the biggest impact on the banking system, but there would have been no capital shortfall (see Figure 26.5). Overall, both satellite models, that is, the IMF satellite model and the structural model, yielded similar and consistent results despite their rather different specifications. They suggested robustness of estimates under either a panel regression approach or a structural approach via updates of model parameters using forecast changes in profitability.

However, the Systemic CCA framework also allowed for the examination of more extreme outcomes amid a rapidly deteriorating macroeconomic environment. The estimation results, which show relatively benign outcomes, pointed to the existing challenges of incorporating the “extreme tail risk” of such a situation. Given that the Systemic CCA framework considers the stochastic nature of risk factors, we examined the realization of a 5 percent “tail of the tail” risk event (as the average density beyond the 95th percentile statistical confidence level) for multiple firms experiencing a dramatic escalation of losses. Under the adverse scenarios, the market-implied capital shortfall would have been significantly increased, likely as a result of significantly lower profitability in conjunction with a sharp deterioration in asset quality and weaker fee-based income reducing the implied asset value of sample banks.

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Note: The multivariate density is generated from univariate marginals that conform to the GEV distribution and a non-parametrically identified time-varying dependence structure. The marginal severity and dependence measure are estimated (i) over a rolling window of 120 working days (with daily updating) for the historical measure (up to a sample cut-off at end-2010) and (ii) over a rolling window of 60 months (with monthly updating) for the generation of forecast values (using the historical dynamics of capital losses as statistical support).

CCA = contingent claims analysis.

**Figure 26.5** United Kingdom: Systemic CCA Estimates of Market-Implied Joint Capital Losses, with IMF Satellite Model (in billions of pound sterling)

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Sample period: 01/03/2005–03/29/2010, monthly observations of historical and forecast joint capital losses of seven sample firms.
Note: The multivariate density is generated from univariate marginals that conform to the GEV distribution and a non-parametrically identified time-varying dependence structure. The marginal severity and dependence measure are estimated (i) over a rolling window of 120 working days (with daily updating) for the historical measure (up to a sample cut-off at end-2010) and (ii) over a rolling window of 60 months (with monthly updating) for the generation of forecast values (using the historical dynamics of capital losses as statistical support).

CCA = contingent claims analysis.
The dispersion of individual contributions to the realization of the central case and extreme tail systemic risk suggests that a few banks contributed disproportionately to joint solvency risk under stress. Table 26.5 shows the percentage share of the minimum, maximum, and interquartile range of the individual banks’ time-varying contribution to the multivariate density of potential losses at the 50th (median) and 95th percentiles. With the exception of the first phase of the European sovereign debt crisis (during most of 2010 and the beginning of 2011), one bank, represented by the maximum of the distribution of all banks, seemed to have accounted for more than half of the solvency risk in the sample.

4. CONCLUSION

The identification of systemic risk is an integral element in the design and implementation of MPS with a view toward enhancing the resilience of the financial sector. However, assessing the magnitude of systemic risk is not a straightforward exercise and has many conceptual challenges. In particular, there is danger of underestimating practical problems of applying systemic risk management approaches to a real-life situation as complex modeling risks lose transparency, with conclusions being highly dependent on the assumptions, the differences in business models of financial institutions, and differences in regulatory and supervisory approaches across countries.

This chapter presented a new modeling framework, Systemic CCA, which can help in the measurement and analysis of systemic solvency risk by estimating the joint expected losses of financial institutions based on multivariate EVT—with practical applications for stress testing. Based on its recent application to the banking sector in the context of the IMF’s FSAP exercise for the United Kingdom (2011), this chapter demonstrates how the individual contributions to systemic risk can be quantified under different stress scenarios and how such systemic relevance of financial institutions can be used to design macroprudential policy instruments, such as systemic risk surcharges. The aggregation model underlying the Systemic CCA framework also was used to illustrate the systemic risk of the financial sector implicitly taken on by governments and the potential (nonlinear) destabilizing feedback processes between the financial sector and the sovereign balance sheet.

Some limitations of the Systemic CCA framework need to be acknowledged and have already been reflected as caveats in the interpretation of the findings. For instance, the market-based determination of default risk might be influenced by the validity of valuation models. Economic models are constructed as general representations of reality that, at best, capture the broad outlines of economic phenomena in a steady state. However, financial market behavior might defy statistical assumptions of valuation models, such as the option pricing theory, extreme value measurement, and nonparametric specification of dependence between individual default probabilities in the Systemic CCA approach. For instance, during the credit crisis, financial market behavior was characterized by rare and nonrecurring events and not by repeated realizations of predictable outcomes generated by a process of random events that exhibits stochastic stability. Thus, the statistical apparatus underlying conventional asset pricing theory fails to fully capture sudden and unexpected realizations beyond historical precedent.

**TABLE 26.5**

United Kingdom: Systemic CCA Estimates of Market-Implied Individual Contributions of Sample Banks to Systemic Risk—Market-Implied Joint Capital Loss (average per time period, in percent of joint capital shortfall)

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>0.1</td>
<td>0.3</td>
<td>0.8</td>
<td>0.6</td>
<td>0.2</td>
<td>1.0</td>
<td>0.4</td>
</tr>
<tr>
<td>25th percentile</td>
<td>2.4</td>
<td>1.5</td>
<td>1.8</td>
<td>1.5</td>
<td>2.1</td>
<td>2.6</td>
<td>2.0</td>
</tr>
<tr>
<td>Median</td>
<td>5.8</td>
<td>4.9</td>
<td>3.1</td>
<td>4.5</td>
<td>6.8</td>
<td>8.1</td>
<td>5.7</td>
</tr>
<tr>
<td>75th percentile</td>
<td>15.3</td>
<td>16.0</td>
<td>6.8</td>
<td>20.0</td>
<td>21.8</td>
<td>22.1</td>
<td>17.2</td>
</tr>
<tr>
<td>Maximum</td>
<td>58.9</td>
<td>59.9</td>
<td>79.0</td>
<td>51.9</td>
<td>45.2</td>
<td>41.4</td>
<td>55.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Range</th>
<th>Expected Joint Capital Shortfall (Median)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>0.1</td>
</tr>
<tr>
<td>25th percentile</td>
<td>3.6</td>
</tr>
<tr>
<td>Median</td>
<td>7.4</td>
</tr>
<tr>
<td>75th percentile</td>
<td>14.0</td>
</tr>
<tr>
<td>Maximum</td>
<td>57.2</td>
</tr>
</tbody>
</table>

Note: Each bank’s percentage share is based on its time-varying contribution to the multivariate density of capital losses at the 50th (median) and 95th percentiles.

CCA = contingent claims analysis.
Going forward, the Systemic CCA framework could be expanded in scale and scope to include other types of risks. For instance, most recently, the Systemic CCA framework has also been adapted to measure systemic liquidity risk by transforming the net stable funding ratio as standard liquidity measure under Basel III into a stochastic measure of aggregate funding risk (see Chapter 27). In addition, given the flexibility of this framework, the financial sector and sovereign risk analysis could be integrated with macro-financial feedbacks in order to design monetary and fiscal policies. Such an approach could inform the calibration of stress scenarios of banking and sovereign balance sheets and the appropriate use of macroprudential regulation. Using an economy-wide Systemic CCA for all sectors, including the financial sector, non-financial corporates, households, and the government, also can provide new measures of economic output—the present value of risk-adjusted GDP (Gray, Jobst, and Malone, 2010).

REFERENCES


Systemic Contingent Claims Analysis


